Basal Area Growth Estimators for Survivor Component: A Quality Control Application

Charles E. Thomas *and* **Francis A. Roesch, Jr.,** *Institute for Quantitative Studies, Southern Forest Experiment Station, USDA Forest Service, New Orleans, LA.*

ABSTRACT. Several possible estimators are available for basal area growth of survivor trees, when horizontal prism (or point) plots (HPP) are remeasured. This study's comparison of three estimators not only provides a check for the estimate of basal area growth but suggests that they can provide a quality control indicator for yield procedures. An example is derived from remeasurements in Alabama for the Southern Forest Experiment Station by Forest Inventory and Analysis. Remeasurements are for 1962–72 and 1972–82. It is suggested that computation of two or perhaps all three of the estimators be routinely incorporated in analysis of remeasured HPP data. Use of the two elemental estimators can provide a quality assurance check on field procedures.

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Estimates of growth from remeasured horizontal prism (or point) plots (HPP) have been widely criticized. Fixed area plots seem to be the popular choice among foresters. However, HPP samples have been defended by some authorities (Furnival 1979, Iles and Beers 1983) and often provide the only plot information available. HPP sampling does impose a requirement that estimation always recognize the unequal probability of selection. The appropriate probability is proportional to the basal area of the sampled tree—i.e., larger trees (or faster growing ones) have higher probability of being sampled and consequently must be weighted inversely with respect to their basal area. Inventory specialists must remember this whenever they compute averages or any other statistic from the HPP sampled trees. Recent research suggests improvements in growth estimation can be made when composite estimators are computed (Van Deusen et al. 1986, Roesch 1988). The very earliest HPP growth estimators suggested are in fact unbiased and relatively efficient, in the statistical sense (Grosenbaugh 1958, Beers and Miller 1964).

Components of growth are described in a number of publications (e.g., Beers and Miller 1964, Martin 1982, Van Deusen et al. 1986). These components normally relate to volume growth, but the same terminology and symbology can be used to represent basal area growth. The components include ingrowth, mortality, cut, and survivor growth. Survivor growth is the growth on trees above some minimum diameter class at both inventories. In a typical remeasurement period of five years, survivor growth contributes more than 75% of the net growth. The following notation describes specific sampling characteristics of survivor trees defined on horizontal points in this paper:

s = a survivor tree sampled in both inventories.

n = a survivor tree sampled only in the remeasurement inventory with an unknown initial diameter that was larger than some minimum at the beginning of the growth period.[1] (See also Figure A1 in the Appendix.)

The n trees have been referred to as nongrowth because of the early confusion between the population and sampling characteristics. For clarity, these definitions exclude from the sample ingrowth and ongrowth trees, i.e., trees that grew across the minimum diameter threshold during the growth period. Growth estimates may be computed from either or both sets of trees. Estimates from one set of trees (n or s) are referred to as *elemental estimators* in this paper. Estimates that involve both types of sample trees are referred to as *composite estimators*.

HISTORICAL APPLICATION

Over the past 25 years, three estimators of volume growth have

[1] Distinguishing these trees from those whose diameter was smaller than the merchantability limit at the prior inventory requires information from supplemental fixed plots, plus good record keeping, or very precise estimates of individual tree initial diameter.

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been employed by the Southern Forest Survey (now called the Forest Inventory and Analysis group, SO-FIA). The first estimator was an end-of-growth-period composite estimator which has been hinted at in the literature, but the theory was not explictly published until recently (Roesch 1988). The second estimator was the traditional Beers and Miller beginning-of-growth-period-based survivor growth estimator. An elemental estimator, it was adopted for use by the SO-FIA in 1980 and first applied to volume growth in Alabama. The third estimator was developed in Van Deusen et al. 1986. It is a composite estimator and is currently employed to obtain volume growth in the SO-FIA. A fourth estimator, originally suggested by Bitterlich, was used as a check for basal area growth in the 1982 inventory. It is an elemental estimator based on the *n* trees. The Van Deusen et al. estimator reduces to the Bitterlich estimator, when the object of estimation is basal area growth and there is no merchantable threshold. Three basal area growth estimators are discussed in this paper: (1) the Beers and Miller survivor growth estimator; (2) the Bitterlich estimator; and (3) the end of growth period (Roesch 1988) estimator.

During the 1962 inventory of Alabama, the Southern Forest Survey began installing point sample plots consisting of 10 satellite sample points. Almost immediately it was discovered that it was imperative to measure a distance to questionable sample trees, to check the calibration of prisms used to select trees, and to calculate a limiting distance diameter factor to assure that these trees were in fact "in." However, the initial inventory proceeded without complete measurements of all tree distances to point center. When inventory CFI plots were remeasured in 1972, distance to all trees was by that time routinely measured to the nearest foot for each point-sampled tree. However, volume growth estimates were computed using the end-of-period growth estimator, thus avoiding the necessity of reestablishing the initial inventory sample trees at the same time an adjustment to the original inventory was made. Subsequent inventories have included the distance measured to the nearest tenth of a foot, giving a quite accurate assessment of the inclusion of each sampled tree.

Figure 1. Forest inventory and analysis units in Alabama, 1982.

During the 1982 Alabama remeasurement inventory, volume growth estimation was revised. The Beers and Miller, stand component, growth estimation scheme was adopted. The basal area growth estimates were checked informally using the third estimator, the Bitterlich estimator, to assure that growth was being estimated with a high degree of precision, but the results were not formally published. Subsequently, the statistical estimator derived by Van Deusen et al. (1986) has been used to obtain an improved composite-estimator for growth for the Southern Station FIA.

Increasing interest in the growth of the southern commercial forest has encouraged a retrospective analysis of Alabama data from the two preceding growth periods—1962–72 and 1972–82. During the analysis phase it was discovered that the three basal area growth estimators mentioned did not agree for the period 1962–72. Using the original distance and diameter data on all survivor trees in the 1972 remeasurement (during which distance to all trees was measured), revised estimates of the total basal area growth for the period were made, presuming that trees did not move between surveys.

The objective of this paper is to make practicing foresters aware that computation of two of the elemental estimators provides an assurance of the quality of growth estimates in an ongoing remeasurement inventory.

METHODS

This analysis will use the results obtained from three survivor growth estimators to provide an indication of the reliability of the sampling and computation processes.

The first growth estimator, which is often referred to as Beers and Miller's estimator, is based on the tree's selection probability at the time of the initial inventory. Because it is based solely on the initial sample survivor trees, we refer to it as an elemental estimator.

An alternative conceptual basis for the estimator is to consider the sampling as if by "fixing the plot size" at the initial inventory. Fixed plot size refers to fixing tree expansion factor of the initial sample trees for the remeasurement period. It is analogous to estimating growth on a fixed area plot, say one-fifth acre, even though a larger plot, one-quarter acre, is overlaid at the time of remeasurement; only trees tallied on the initial plot would contribute to the growth computation. In other words, for growth estimation the plot size has remained unchanged, one-fifth acre.

The estimator for survivor basal area growth per acre (SG_1) is given by:

$$SG_1 = k(BAF) \sum_{i \epsilon s} (d_{2i}^2 - d_{1i}^2)/ba_{1i} \quad (1)$$

where

k = 0.005454 for a single-point plot,

d_{2i} = final diameter of survivor trees,

d_{1i} = initial diameter of the same set of survivors,

ba_{1i} = survivor trees initial basal area per acre,

$i \epsilon s$ = tree i is an element in set s, survivor trees.

Note the estimator is in terms of per plot growth, not the average per acre.

The second estimator is one that has been attributed to Bitterlich. It is also an elemental estimator rather than composite. Basal area growth for this estimator is quite simply given by:

$$SG_2 = \sum (BAF)m_n \quad (2)$$

where

m_n = the number of nongrowth (n) trees at each point,

BAF = the basal area factor.

It is also noteworthy that the estimator is discrete, each new sample tree adding exactly BAF to the survivor growth estimate.

For all growth estimators, it is important to distinguish among the several possible sample tree classes at all steps in the remeasurement inventory (Martin 1982). Van Deusen et al. (1986) point out that nongrowth n trees are in fact surviving trees, alive, larger than the minimum diameter, but not sampled at the last occasion. Ingrowth and ongrowth trees were previously smaller than the diameter inclusion limit; ongrowth was not previously sampled. Only the former tree (nongrowth) qualifies for inclusion in the summation for equation (2).

The third estimator for basal area growth may be less well known. It is a composite estimator; Roesch (1988) gives details of its theoretical development for both volume and basal area growth. The volume analogue was applied by the Southern Station FIA survey for growth estimation as described earlier. It is based on the correct terminal inventory selection probability or equivalently, "fixing the plot size" at the end of the survey period. It has been described, albeit vaguely, in the literature by a variety of authors (e.g., Iles 1981). Basal area growth per acre of all survivor trees is calculated based on the tally at both measurements. It is dependent on the estimation of basal area growth (or diameter growth) for the newly selected prism trees (the n trees).

The estimator may be expressed:

$$SG_3 = kBAF \left\{ \sum_{i \epsilon s} (d_{2i}^2 - d_{1i}^2)/ba_{2i} + \sum_{j \epsilon n} (d_{2j}^2 - \hat{d}_{1j}^2)/ba_{2j} \right\} \quad (3)$$

where

$i \epsilon s$ = tree i sampled in both inventories,

$j \epsilon n$ = tree j sampled in the final inventory only,

$\hat{d}_{1j}^2$ = estimate of the initial diameter squared.

The end-of-period estimator requires this estimate of initial basal area (diameter) for all n trees. The initial basal area can be obtained from the regression of initial basal area on final basal area for the survivor s type trees. This may entail breaking samples down into species or species groups and Roesch (1988) shows that weighted regressions are necessary to obtain unbiased estimates.

RESULTS

Each of the three estimators was computed for each of the six

Table 1a. Total survivor basal area increment computed for original 1962–72 remeasurement of Alabama forest HPP sample.[1]

Survey region	Estimator		
	Bitterlich	Beers and Miller	End-of-period
	(ft²)		
Southwest-south	3052	2375	2441
Southwest-north	3217	2644	2636
Southeast	9337	5338	6508
West Central	4545	2881	3195
North Central	6412	4069	4598
North	1395	910	968
State	27958	18217	20346

Table 1b. Test for significant differences between Bitterlich and Beers Miller estimates for period 1962–1972 data.

Survey region	Number of plots	Statistic t calculated	t tabulated[2] $P = 0.01$
	(value)		
Southwest-south	135	2.07 ns	2.617
Southwest-north	113	2.72*	2.617
Southeast	269	9.73*	2.576
West Central	129	4.64*	2.617
North Central	199	7.03*	2.576
North	43	3.49*	2.704

[1] The estimates in tables are totals for sampled plots. Forest acreage expansion has not been applied. This eliminates the confounding differences in estimates due to changing acreage between periods, leaving only the basal area growth on the sampled plots.
[2] Tabulated t values are approximated from Freese (1967).

survey regions in the state (Figure 1). The results of this analysis are presented in Tables 1a–b. Differences between the estimators are large, ranging from +22% in the Southwest-North to +44% in the North for the Bitterlich type estimator and from −0.3 to −18% for the Beers-Miller type estimator, given the end-of-period estimator as a base. Statewide the estimators were +37% and −11% when compared to the end-of-period estimator.

Analysis of survey records by the lead author strongly suggests that too few "in" trees were recorded for the Southeast, North Central, West Central, and North survey regions of the state during the 1962 inventory. Because of this, we believe, the Beers-Miller estimate of basal area growth underestimates the actual growth for the period 1962–72. [Recall that the end-of-period estimator of volume growth was employed at the time of the 1972 report and hence the volume growth results from that time were accurate. Any re-analysis of volume growth using Beers and Miller's estimator would only be accurate if the original data were somehow corrected to give initial basal areas for trees miscounted in the initial (1962) inventory.]

The 1972 remeasurement data were reprocessed using the measured distance to the tree for the 1972 survey to determine if each newly sampled tree was in fact new or a "missed" tree from the 1962 survey. After the new determination of the inclusion probability was made, the growth analyses for each unit and the state were repeated. Results of the analysis are presented in Tables 2a–b. At the state level growth differences for the two estimators in the 1972 remeasurement were reduced from +37 and −11 to +4 and −1% for Bitterlich and Beers-Miller, respectively. Similarly, discrepancies between the estimators in each unit were reduced with the exception of the Beers-Miller estimates for the Southwest units, in which relatively small negative differences were switched to small positive differences.

The 1982 (for the period 1972–82) remeasurement data were also computed for all survey regions in the State. The results of this analysis are presented in Tables 3a–b. This later survey indicates that the two elemental growth estimators are quite consistent. There are no statistically significant differences between the Bitterlich and Beers-Miller estimators. Differences between the end-of-period and Beers-Miller estimator are larger, but only one

Table 2a. Total survivor basal area increment computed after distance to tree adjustment to tree history is made.

Survey region	Estimator		
	Bitterlich	Beers and Miller	End-of-period
	(ft²)		
Southwest-south	2411	2539	2441
Southwest-north	2610	2743	2636
Southeast	6888	6326	6508
West Central	3386	3183	3195
North Central	4800	4447	4598
North	1057	960	968
State	21152	20198	20346

Table 2b. Test for significant differences between Bitterlich and Beers Miller estimates for adjusted 1962–1972 data.

Survey region	Number of plots	Statistic t calculated	t tabulated $P = 0.01$
	(value)		
Southwest-south	135	0.84 ns	2.617
Southwest-north	113	0.83 ns	2.617
Southeast	269	2.15 ns	2.576
West Central	129	1.04 ns	2.617
North Central	199	1.77 ns	2.576
North	43	1.17 ns	2.704

unit appeared to have a difference that could be statistically significant. The initial diameters were not estimated specifically for this analysis. There is good reason to believe that the initial diameter estimates for some species groups were biased. While the end-of-period estimator computed here should not be relied on as the sole estimate of growth it provides a helpful reference should the two principal estimators fail. Theoretical developments (Roesch 1988) suggest that a reliable composite estimator essentially similar to the end-of-period basal area growth estimator, can be computed if properly weighted diameter growth regressions are employed to obtain estimates of the initial diameter for n type trees.

DISCUSSION

The analyses of these historical Southern Forest Station survey data have suggested some useful results for the estimation of growth from HPP. In many instances, generalization from the extensive multipurpose FIA inventories to other types of forest inventory are not warranted. However, the body of information concerning growth from permanent HPP samples has been consolidated in the 1980s and, with extensive computing capabilities now generally available, these results may easily be implemented for other forest inventories.

The theory of remeasured HPP has recently received useful modification in Van Deusen et al. (1986) and subsequently in Roesch et al. (1989). Composite estimators for volume growth have been developed. Application of composite estimators for growth can give reductions in bias and in mean squared error of the estimators, which make them worthwhile in the determination of volume or basal area growth.

Using the current field procedures and examining all available growth estimators could provide important cross-checks on the estimate of basal area growth. No change in field procedures should be needed other than measuring distance to each sample tree, where that has not been done in the past. Now, all of the information in the field data can be utilized efficiently after separately computing the three estimates. Two of the estimators (SG_s and SG_n) are computed from distinct sets of trees. These trees are not strictly independent in a statistical sense, but they do provide a degree of independence in the computation of growth. The third estimator depends on the precision of the estimate of initial diameter for the set of nongrowth (n) trees. Overestimation of the diameter growth will result in an overestimate of growth with this estimator; hence it is important to apply the correctly specified weighted regression model to determine initial diameter for these trees.

Table 3a. Total survivor basal area increment computed for 1972–82 (2nd) remeasurement of Alabama forest HPP samples.

	Estimator		
Survey region	Bitterlich	Beers and Miller	End-of-period
	(ft²)		
Southwest-south	2572	2656	2837
Southwest-north	4665	4239	4764
Southeast	6112	6045	6352
West Central	3862	3713	3984
North Central	5445	5647	5968
North	1237	1114	1197
State	23893	23414	25102

Table 3b. Test for significant differences between Bitterlich and Beers Miller estimates for 1972–1982 data.

Survey region	Number of plots	Statistic t calculated	t tabulated $P = 0.01$
	(value)		
Southwest-south	149	0.38 ns	2.617
Southwest-north	158	1.42 ns	2.617
Southeast	257	0.19 ns	2.576
West Central	134	0.55 ns	2.617
North Central	193	0.40 ns	2.576
North	43	0.78 ns	2.704

Quality Control

During the process of the remeasurement inventory, estimates of the basal area from both estimators (1) and (2) can be easily computed. Since the expected values of these two estimators are equal (Appendix A) and the trees represent disjoint sets, we propose that they may provide a quality monitoring capability during ongoing remeasurement inventories. Computing the two estimates during the data collection process could help detect field blunders such as the inadvertent introduction of a miscalibrated prism. They also provide a cross-check on the computational procedures used in a large inventory. Computing these two basic estimators provides independent checks on the growth, an advantage not possessed by fixed area growth plots.

It is important that practicing inventory foresters recognize the selection of a BAF appropriate to the expected growth per acre for the growth period is critical to the process. If the periodic basal area growth is very small and the BAF employed in the inventory is large, then the estimate of basal area growth from the Bitterlich estimator will be extremely variable. In this case it is unlikely that computing the two estimators would be very efficient. If the growth period is short, then smaller factor prisms will be required in order to detect the change. A second consideration is the size of the trees being inventoried. If the trees are on average very large, then the Bitterlich estimator would again be quite variable, because the estimator only detects a whole tree. In the relatively intensively managed and rapidly growing Southern Forest this latter condition should rarely occur.

CONCLUSION

Three growth estimators for use with HPP samples have been presented. Based on differences in the original data, edit checks for the distance to point center for new and old trees on remeasured plots could provide a valuable check on field work. Computation of all three growth estimates is readily implemented in ongoing inventories. Comparison of the two principal estimates (Beers and Miller versus modified Bitterlich) provides a positive opportunity to assess the reliability of the growth estimates for survivor trees. The comparison also affords an inventory group with a ready means to check the progress and quality of ongoing inventory field work that is designed to measure basal area growth.

A note of warning needs to be appended, use of the Bitterlich estimator for volume growth could be seriously biased. It is quite dependent on changes in the expected value of volume to basal area ratio, which can change systematically with the maturation of the forest. It can also be changed dramatically by the timing of harvest in an inventory unit. Monitoring the volume to basal area ratio should be considered by inventory groups concerned with the growth of stands for which they are responsible. □

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Appendix A

The following gives a brief mathematical treatment of the equivalence in expectation of the sum for the two basal area growth estimators for the survivor component. It parallels the development of the expected value for survivor growth given in Van Deusen et al. (1986).

Figure A1 illustrates the selection circle for trees that grow during a survey period. The inner circle represents the initial selection probability and is proportion to the initial basal area (b_{1i}). The outer circle represents the selection probability at the end of the period and is proportional to the final basal area (b_{2i}). The annulus, denoted n, represents the growth of the tree during the period. A merchantable tree is an s tree if the point is located in the inner circle in the initial inventory. A tree's annulus that overlaps the point at the end of the growth (inter-inventory) period is an n tree. The expected value for growth from n trees $E(SG_n)$ can be expressed by:

$$\begin{aligned} E(SG_n) &= E\left(BAF \sum_{i \in M} \frac{b_{2i}}{b_{2i}} I_{n_i}\right) \\ &= BAF \sum_{i \in M} \frac{b_{2i}}{b_{2i}} \mathrm{E}(I_{n_i}) \end{aligned} \qquad \text{(A1)}$$

where M indicates the population of merchantable trees. Obviously the fraction b_{2i}/b_{2i} is just the count of trees, but is given in this form for consistency of expression of the basal area selection probability. Further,

$$E(I_{n_i}) = \frac{b_{2i} - b_{1i}}{BAF} \qquad \text{(A2)}$$

gives the expected value for nongrowth trees selected at the end of the period, and where I_n is 1 for tree i if it is selected as a survivor (n) trees only, 0 otherwise.

Then substituting A2 into A1 gives:

$$E(SG_n) = \sum_{i \in M} (b_{2i} - b_{1i}) \qquad \text{(A3)}$$

The expected value for growth on s trees $E(SG_s)$ can be written:

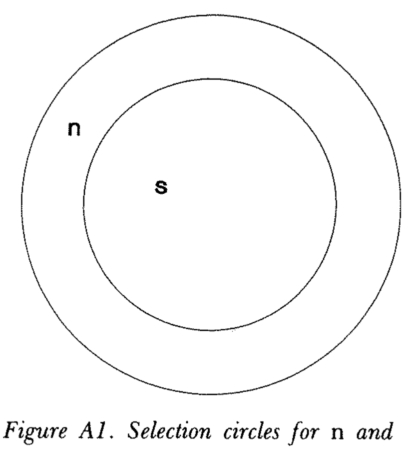

Figure A1. Selection circles for n *and* s *type trees.*

$$E(SG_s) = E\left(BAF \sum_{i \epsilon M} \frac{b_{2i} - b_{1i}}{b_{1i}} I_{s_i}\right) \quad (A4)$$

where M is again the population of merchantable trees and expectation reduces to the expected value for the indicator as before

$$E(I_{s_i}) = \frac{b_{1i}}{BAF} \quad (A5)$$

gives the expected value for survivor trees selected in the beginning of the inventory period, and where I_s is 1 for tree i if it is selected as a survivor (s) trees only, 0 otherwise.

Substitution yields

$$E(SG_s) = \sum_{i \epsilon M} (b_{2i} - b_{1i})$$

and therefore

$$E(SG_s) = E(SG_n).$$